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RESEARCH ARTICLE

Edge-vertex dominating sets and edge-vertex domination polynomials of centipedes

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Abstract

Let $G = (V, E)$ be a simple graph. A set $S \subseteq E(G)$ is an edge-vertex dominating set of G (or simply an *ev*-dominating set), if for all vertices $v \in V(G)$, there exists an edge $e \in S$ such that e dominates v. Let P_n^* be the centipede corresponding to the path P_n and let $D_{ev}(P_n^*, i)$ denote the family of all *ev*-dominating sets of P_n^* with cardinality *i*. Let $d_{ev}(P_n^*, i) = |D_{ev}(P_n^*, i)|$. In this paper, we obtain a recursive formula for $d_{ev}(P_n^*, i)$. Using this recursive formula, we construct the Polynomial, $D_{ev}(P_n^*, x) = \sum_{i=\lceil \frac{n}{2} \rceil}^{n} d_{ev}(P_n^*, i) x^i$, which we call Edge-Vertex Domination polynomial of P_n^* and obtain some properties of edge-vertex dominating sets of centipedes.

Keywords

Edge-Vertex Dominating Sets

Edge-Vertex Domination Number

Edge-Vertex Domination Polynomials

1. Introduction

Let $G = (V, E)$ be a simple graph of order $|V| = n$. A set $S \subseteq V(G)$ is a dominating set of G, if every vertex $V \setminus S$ is adjacent to at least one vertex in S. For any vertex $v \in V$, the open neighbourhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighbourhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighbourhood of S is $N(S) = U_{v \in S} N(v)$ and the closed neighbourhood of S is $N[S] = N(S) \cup S$. The domination number of a graph G is defined as the minimum size of a dominating set in G and it is denoted as $\gamma(G)$. A Path is a connected graph in which end vertices have degree one and the remaining vertices have degree two, and is denoted by P_n. The centipede P_n^* consists of a path P_n in which each vertex imbedded with a pendant edge and a pendant vertex.

Definition 1.1

For a graph $G = (V, E)$, an edge $e = uv \in E(G)$, *ev*-dominates a vertex $w \in V(G)$ if

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(i) $u = w$ or $v = w$ (w is incident to e) or

(ii) uw or vw is an edge in G (w is adjacent to u or v).

Definition 1.2[1]

A set $S \subseteq E(G)$ is an edge-vertex dominating set of G (or simply an *ev*-dominating set), if for all vertices $v \in V(G)$, there exist an edge $e \in S$ such that e dominates v. The *ev*-domination number of a graph G is defined as the minimum size of an *ev*-dominating set of edges in G and it is denoted as $\gamma_{ev}(G)$.

Definition 1.3[2]

Let $D_{ev}(P_n^*, i)$ be the family of *ev*-dominating sets of a centipede graph P_n^* with cardinality i and let $d_{ev}(P_n^*, i) = |D_{ev}(P_n^*, i)|$. We call the polynomial $D_{ev}(P_n^*, x) = \sum_{i=\lceil \frac{n}{2} \rceil}^{n} d_{ev}(P_n^*, i) x^i$ the *ev*- domination polynomial of the graph P_n^*. As usual we use $\lfloor x \rfloor$ for the largest integer less than or equal to x and $\lceil x \rceil$ for the smallest integer greater than or equal to x. Also, we denote the set $\{e_1, e_2, \cdots, e_n\}$ by [e_n] and the set $\{1, 2, \cdots, n\}$ by [n], throughout this paper.

2. Edge-vertex dominating sets of centipedes

Lemma 2.1:[3] $\gamma_{ev}(P_n) = \left\lceil \frac{n}{4} \right\rceil$.

By Lemma 2.1 and the definition of *ev*-domination number, one has the following Lemma:

Lemma 2.2.[4]

For every $n \in N$,

i) $\gamma_{ev}(P_n^*) = \left\lceil \frac{n}{2} \right\rceil$

ii) $\gamma_{ev}(P_n^* - \{e_{2n-1}\}) = \left\lfloor \frac{n}{2} \right\rfloor$

iii) $D_{ev}(P_n^*, i) = \Phi$ if and only if $i < \left\lceil \frac{n}{2} \right\rceil$ or $i > 2n-1$

iv) $D_{ev}(P_n^* - \{e_{2n-1}\}, i) = \Phi$ if and only if $i < \left\lfloor \frac{n}{2} \right\rfloor$ or $i > 2n-2$

Proof:

(i) Clearly, if n is even $\{e_2, e_6, e_{10}, ..., e_{2n-2}\}$ is a minimum ev-dominating set for P_n^* and if n is odd $\{e_1, e_4, e_8, ..., e_{2n-2}\}$ is one of the minimum ev-dominating set for P_n^*. If n is even or odd it contains $\left\lceil \frac{n}{2} \right\rceil$ elements. Hence $\gamma_{ev}(P_n^*) = \left\lceil \frac{n}{2} \right\rceil$.

(ii) Clearly, if n is even $\{e_2, e_4, ..., e_{2n-2}\}$ is one of the minimum *ev*-dominating set for $P_n^* - \{e_{2n-1}\}$ and if n is odd $\{e_2, e_6, ..., e_{2n-4}\}$ is the minimum *ev*-dominating set for $P_n^* - \{e_{2n-1}\}$. If n is even or odd it contains $\left\lfloor \frac{n}{2} \right\rfloor$ elements. Hence $\gamma_{ev}(P_n^* - \{e_{2n-1}\}) = \left\lfloor \frac{n}{2} \right\rfloor$.

(iii) It follows from (i) and the definition of *ev*-dominating set.

(iv) It follows from (ii) and the definition of *ev*-dominating set.For the construction of $D_{ev}(P_n^*, i)$ we consider $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)$, $D_{ev}(P_{n-1}^*, i-1)$ and $D_{ev}(P_{n-2}^*, i-1)$.

Lemma 2.3[5]

i) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-2}^*, i-1) = \Phi$, then $D_{ev}(P_{n-1}^*, i-1) = \Phi$.

ii) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \neq \Phi$ and $D_{ev}(P_{n-2}^*, i-1) \neq \Phi$, then $D_{ev}(P_{n-1}^*, i-1) \neq \Phi$.

iii) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-1}^*, i-1) = D_{ev}(P_{n-2}^*, i-1) = \Phi$, then $D_{ev}(P_n^*, i) = \Phi$.

iv) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-1}^*, i-1) = \Phi$ and $D_{ev}(P_{n-2}^*, i-1) \neq \Phi$, then $D_{ev}(P_n^*, i) \neq \Phi$.

Proof:

i) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-2}^*, i-1) = \Phi$, by Lemma 2.2 (iii), $i-1 < \left\lfloor \frac{n}{2} \right\rfloor$ or $i-1 > 2n-2$ and $i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ or $i-1 > 2n-5$. Therefore, $i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ or $i-1 > 2n-2$. Therefore, $i-1 < \left\lceil \frac{n-1}{2} \right\rceil$ or $i-1 > 2n-3$. Therefore, $D_{ev}(P_{n-1}^*, i-1) = \Phi$.

ii) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \neq \Phi$, then by lemma 2.2(iii) we have, $i-1 \geq \left\lfloor \frac{n}{2} \right\rfloor$ and $i-1 \leq 2n-2$

If $D_{ev}(P^*_{n-2}, i-1) \neq \Phi$, then by lemma 2.2 (iv) we have, $i-1 \geq \left\lceil \frac{n-2}{2} \right\rceil$ and $i-1 \leq 2n-5$. Therefore, $\left\lfloor \frac{n}{2} \right\rfloor \leq i-1 \leq 2n-5$. Since, $i-1 \leq 2n-2$, which implies $i-1 \leq 2n-3$ and $i-1 \geq \left\lceil \frac{n-2}{2} \right\rceil$, which implies $i-1 \geq \left\lceil \frac{n-1}{2} \right\rceil$. Therefore, $i-1 \geq \left\lceil \frac{n-1}{2} \right\rceil$ and $i-1 \leq 2n-3$. Therefore, $D_{ev}(P^*_{n-1}, i-1) \neq \Phi.$

iii) If $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) = D_{ev}(P^*_{n-1}, i-1) = D_{ev}(P^*_{n-2}, i-1) = \Phi,$ then by lemma 2.2 (iii) and (iv) we have, $i-1 < \left\lfloor \frac{n}{2} \right\rfloor$ or $i-1 > 2n-2$, $i-1 < \left\lceil \frac{n-1}{2} \right\rceil$ or $i-1 > 2n-3$ and $i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ or $i-1 > 2n-5$. Hence, $i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ or $i-1 > 2n-2$. Therefore, $i < \left\lceil \frac{n-2}{2} \right\rceil + 1$ or $i > 2n-1$. Therefore, $i < \left\lceil \frac{n}{2} \right\rceil$ or $i > 2n-1$. Therefore, $D_{ev}(P^*_n, i) = \Phi.$

iv) If $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) = D_{ev}(P^*_{n-1}, i-1) = \Phi$ then by Lemma 2.2 (iii) and (iv) we have, $i-1 < \left\lfloor \frac{n}{2} \right\rfloor$ or $i-1 > 2n-2$, $i-1 < \left\lceil \frac{n-1}{2} \right\rceil$ or $i-1 > 2n-3$. If $D_{ev}(P^*_{n-2}, i-1) \neq \Phi,$ by Lemma 2.2 (iii) we have, $i-1 \geq \left\lceil \frac{n-2}{2} \right\rceil$ and $i-1 \leq 2n-5$. Since $i-1 \leq 2n-5$, $i-1 > 2n-3$ is not possible. Hence, $i-1 \geq \left\lceil \frac{n-2}{2} \right\rceil$ and $i-1 \leq 2n-2$ hold. Therefore, $i \geq \left\lceil \frac{n-2}{2} \right\rceil + 1$ and $i \leq 2n-1$. Therefore, $i \geq \left\lceil \frac{n}{2} \right\rceil$ and $i \leq 2n-1$. Therefore, $D_{ev}(P^*_n, i) \neq \Phi.$

Theorem 2.4[6]

If $D_{ev}(P^*_n, i) \neq \Phi,$ then

i) $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) \neq \Phi$ and $D_{ev}(P^*_{n-1}, i-1) = D_{ev}(P^*_{n-2}, i-1) = \Phi$ if and only if

ii) $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) \neq \Phi, D_{ev}(P^*_{n-1}, i-1) \neq \Phi$ and $D_{ev}(P^*_{n-2}, i-1) = \Phi,$ if and only if $i = 2n-3$ or $i = 2n-2$.

iii) $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) \neq \Phi, D_{ev}(P^*_{n-1}, i-1) \neq \Phi, D_{ev}(P^*_{n-2}, i-1) \neq \Phi$, if and only if $\left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq 2n-4$.

iv) $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) = D_{ev}(P^*_{n-1}, i-1) = \Phi$ and $D_{ev}(P^*_{n-2}, i-1) \neq \Phi,$ if and only if $n = 2k$ and $i = k$ for some $k \in N$.

Proof:

i) $(\Rightarrow)$ Since $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) \neq \Phi$ and $D_{ev}(P^*_{n-1}, i-1) = D_{ev}(P^*_{n-2}, i-1) = \Phi$ by lemma 2.2 (iii) and (iv) we have, $i-1 \geq \left\lfloor \frac{n}{2} \right\rfloor$ and

$$i-1 \leq 2n-2 \quad (2.1)$$

$$i-1 < \left\lceil \frac{n-1}{2} \right\rceil \text{ or } i-1 > 2n-3 \quad (2.2)$$

$$i-1 < \left\lceil \frac{n-2}{2} \right\rceil \text{ or } i-1 > 2n-5 \quad (2.3)$$

Hence, from (2.2) and (2.3) we have, $i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ or $i-1 > 2n-3$. By (2.1) $i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ is not possible. Therefore, $i-1 > 2n-3$. Therefore, $i-1 \geq 2n-2$
Therefore, $i \geq 2n-1$ (2.4)

But, $i-1 \leq 2n-2$. Therefore, $i \leq 2n-1$. (2.5)
From (2.4) and (2.5), $i = 2n-1$.

$(\Leftarrow)$ If $i = 2n-1$, $i-1 = 2n-2$ then by lemma 2.2 (iii) and (iv), $i-1 > 2n-3$.

Therefore, $D_{ev}(P^*_{n-1}, i-1) = \Phi$. And $i-1 > 2n-5$. Therefore, $D_{ev}(P^*_{n-2}, i-1) = \Phi$. Also we have, $\left\lfloor \frac{n}{2} \right\rfloor \leq i-1 < 2n-1$. Therefore, $\left\lfloor \frac{n}{2} \right\rfloor \leq i-1 \leq 2n-2$. Therefore, $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) \neq \Phi.$

ii) $(\Rightarrow)$Since $D_{ev}(P^*_n - \{e_{2n-1}\}, i-1) \neq \Phi, D_{ev}(P^*_{n-1}, i-1) \neq \Phi$ and $D_{ev}(P^*_{n-2}, i-1) = \Phi,$ by lemma 2.2 (iii) and (iv) we have,

$$\left\lfloor \frac{n}{2} \right\rfloor \leq i-1 \leq 2n-2 \quad (2.6)$$

$$\left\lceil \frac{n-1}{2} \right\rceil \leq i-1 \leq 2n-3 \quad (2.7)$$

$$\text{And } i-1 < \left\lceil \frac{n-2}{2} \right\rceil \text{ or } i-1 > 2n-5 \quad (2.8)$$

From (2.6) and (2.7), we have

$$\left\lfloor \frac{n}{2} \right\rfloor \le i-1 \le 2n-3 \qquad (2.9)$$

$i-1 < \left\lceil \frac{n-2}{2} \right\rceil$ is not possible. Therefore, $i-1 > 2n-5$. Therefore, $i > 2n-4$. Therefore, $i \ge 2n-3$. But, $i-1 \le 2n-3$. Therefore, $i = 2n-3$ or $i = 2n-2$.

(⇐) Assume $i = 2n-3$, $i-1 = 2n-4$ or $i = 2n-2$, $i-1 = 2n-3$

Case (i): If $i-1 = 2n-4$ then by lemma 2.2 (iii) and (iv), $i-1 \ge \left\lfloor \frac{n}{2} \right\rfloor$ and $i-1 \le 2n-2$. Therefore, $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \ne \Phi$ and $i-1 \ge \left\lceil \frac{n-1}{2} \right\rceil$ and $i-1 \le 2n-3$. Therefore, $D_{ev}(P_{n-1}^*, i-1) \ne \Phi$. Also, $i-1 = 2n-4 > 2n-5$. Therefore, $D_{ev}(P_{n-2}^*, i-1) = \Phi$.

Case (ii): If $i-1 = 2n-3$ then by lemma 2.2 (iii) and (iv), $i-1 \ge \left\lfloor \frac{n}{2} \right\rfloor$ and $i-1 \le 2n-2$. Therefore, $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \ne \Phi$ and $i-1 \ge \left\lceil \frac{n-1}{2} \right\rceil$ and $i-1 \le 2n-3$. Therefore, $D_{ev}(P_{n-1}^*, i-1) \ne \Phi$. Also, $i-1 = 2n-3 > 2n-5$. Therefore, $D_{ev}(P_{n-2}^*, i-1) = \Phi$.

iii) (⇒) Since $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \ne \Phi$, $D_{ev}(P_{n-1}^*, i-1) \ne \Phi$ and $D_{ev}(P_{n-2}^*, i-1) \ne \Phi$ by lemma 2.2 (iii) and (iv), $\left\lfloor \frac{n}{2} \right\rfloor \le i-1 \le 2n-2$, $\left\lceil \frac{n-1}{2} \right\rceil \le i-1 \le 2n-3$ and $\left\lceil \frac{n-2}{2} \right\rceil \le i-1 \le 2n-5$.

So, $\left\lfloor \frac{n}{2} \right\rfloor \le i-1 \le 2n-5 \Rightarrow \left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le 2n-4$.

(⇐) If $\left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le 2n-4$, then by lemma 2.2 (iii) we have, $\left\lfloor \frac{n}{2} \right\rfloor \le i-1 \le 2n-2$, $\left\lceil \frac{n-1}{2} \right\rceil \le i-1 \le 2n-3$ and $\left\lceil \frac{n-2}{2} \right\rceil \le i-1 \le 2n-5$.

Therefore, $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \ne \Phi, D_{ev}(P_{n-1}^*, i-1) \ne \Phi,$ and $D_{ev}(P_{n-2}^*, i-1) \ne \Phi$

(iv) (⇒) Since $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-1}^*, i-1) = \Phi$, by lemma 2.2 (iii) and (iv) we have, $i-1 < \left\lfloor \frac{n}{2} \right\rfloor$ or $i-1 > 2n-2$, $i-1 < \left\lceil \frac{n-1}{2} \right\rceil$ or $i-1 > 2n-3$.

Hence, $i-1 < \left\lceil \frac{n-1}{2} \right\rceil$ or $i-1 > 2n-2$. If $i-1 > 2n-2$, then $i > 2n-1$ and by lemma 2.2 (iii), $D_{ev}(P_n^*, i) = \Phi$, a contradiction. So,

$$i-1 < \left\lceil \frac{n-1}{2} \right\rceil \qquad (2.10)$$

And since $D_{ev}(P_{n-2}^*, i-1) \ne \Phi$ we have,

$$\left\lceil \frac{n-2}{2} \right\rceil \le i-1 \le 2n-5 \qquad (2.11)$$

From (2.10) and (2.11), $\left\lceil \frac{n-2}{2} \right\rceil \le i-1 < \left\lceil \frac{n-1}{2} \right\rceil$.

When n is a multiple of 2, $\left\lceil \frac{n-2}{2} \right\rceil = \frac{n}{2} - 1$ and $\left\lceil \frac{n-1}{2} \right\rceil = \frac{n}{2}$. Therefore, $\frac{n}{2} - 1 \le i-1 < \frac{n}{2}$. Therefore, $i-1 = \frac{n}{2} - 1$ we get, $i = \frac{n}{2}$. Thus, when $n = 2k$, (2.11) holds good and $i = \frac{n}{2} = k$. When $n \ne 2k$, $\left\lceil \frac{n-2}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil - 1$ and $\left\lceil \frac{n-1}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil - 1$. Therefore, $\left\lceil \frac{n}{2} \right\rceil - 1 \le i-1 < \left\lceil \frac{n}{2} \right\rceil - 1$, which is not possible. Hence, $n = 2k$ and $i = k$.

(⇐) If $n = 2k$ and $i = k$ for some $k \in N$, then by lemma 2.2, $\left\lceil \frac{n-1}{2} \right\rceil = \left\lceil \frac{2k-1}{2} \right\rceil = k = i > i-1$

Therefore, $D_{ev}(P_{n-1}^*, i-1) = \Phi$. And $\left\lfloor \frac{n-1}{2} \right\rfloor = \left\lfloor \frac{2k-1}{2} \right\rfloor = k-1 = i-1$. Therefore, $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = \Phi$. Now, $\left\lceil \frac{n-2}{2} \right\rceil = \left\lceil \frac{2k-2}{2} \right\rceil = k-1 = i-1$. Therefore, $\left\lceil \frac{n-2}{2} \right\rceil \le i-1$ which implies $D_{ev}(P_{n-2}^*, i-1) \ne \Phi$.

Theorem 2.5[7]

i) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \neq \Phi$, $D_{ev}(P_{n-1}^*, i-1) = \Phi$ and $D_{ev}(P_{n-2}^*, i-1) = \Phi$ then $D_{ev}(P_n^*, i) = \{X \bigcup \{e_{2n-1}\} / X \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\}$.

ii) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-1}^*, i-1) = \Phi$ and $D_{ev}(P_{n-2}^*, i-1) \neq \Phi$ then $D_{ev}(P_n^*, i) = \{X \bigcup \{e_{2n-2}\} / X \in D_{ev}(P_{n-2}^*, i-1)\}$.

iii) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_{n-1}^*, i-1) \neq \Phi$ and $D_{ev}(P_{n-2}^*, i-1) = \Phi$ then

$$D_{ev}(P_n^*, i) = \left\{ \begin{array}{l} \{X_1 \bigcup \{e_{2n-1}\} / X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\} \bigcup \\ \{X_2 \bigcup \{e_{2n-2}\} / X_2 \in D_{ev}(P_{n-1}^*, i-1)\} \end{array} \right\}.$$

iv) If $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) \neq \Phi$, $D_{ev}(P_{n-1}^*, i-1) \neq \Phi$ and $D_{ev}(P_{n-2}^*, i-1) \neq \Phi$ then

$$D_{ev}(P_n^*, i) = \left\{ \begin{array}{l} \{X_1 \bigcup \{e_{2n-1}\} / X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\} \bigcup \\ \{X_2 \bigcup \{e_{2n-2}\} / X_2 \in D_{ev}(P_{n-1}^*, i-1)\} \bigcup \\ \{X_3 \bigcup \{e_{2n-2}\} / X_3 \in D_{ev}(P_{n-2}^*, i-1)\} \end{array} \right\}$$

Proof:

i) By theorem 2.4 (i), $i = 2n-1$. Since in this case $D_{ev}(P_n^*, i) = D_{ev}(P_n^*, 2n-1) = \{[e_{2n-1}]\}$ and $D_{ev}(P_n^* - \{e_{2n-1}\}, i-1) = D_{ev}(P_n^* - \{e_{2n-1}\}, 2n-1) = \{[e_{2n-2}]\}$, then we have the result.

ii) Let $Y_1 = \{X \bigcup \{e_{2n-2}\} / X \in D_{ev}(P_{n-2}^*, i-1)\}$. Obviously $Y_1 \subseteq D_{ev}(P_n^*, i)$. Now, let $Y \in D_{ev}(P_n^*, i)$ then $e_{2n-2} \in Y$. If $e_{2n-2} \in Y$, then we can write $Y = X \bigcup \{e_{2n-2}\}$, for some $X \in D_{ev}(P_{n-2}^*, i-1)$, that is $Y \in Y_1$. Thus we have proved that, $D_{ev}(P_n^*, i) \subseteq Y_1$. Hence $D_{ev}(P_n^*, i) = \{X \bigcup \{e_{2n-2}\} / X \in D_{ev}(P_{n-2}^*, i-1)\}$.

iii) Let $Y_1 = \{X_1 \bigcup \{e_{2n-1}\} / X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\}$ and $Y_2 = \{X_2 \bigcup \{e_{2n-2}\} / X_2 \in D_{ev}(P_{n-1}^*, i-1)\}$. Obviously, $Y_1 \bigcup Y_2 \subseteq D_{ev}(P_n^*, i)$. Now, Let $Y \in D_{ev}(P_n^*, i)$ then e_{2n-1} or $e_{2n-2} \in Y$. If $e_{2n-1} \in Y$, then we can write $Y = X_1 \bigcup \{e_{2n-1}\}$, for some $X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)$, that is $Y \in Y_1$. If $e_{2n-1} \notin Y$ and $e_{2n-2} \in Y$, then we can write $Y = X_2 \bigcup \{e_{2n-2}\}$, for some $X_2 \in D_{ev}(P_{n-1}^*, i-1)$, that is $Y \in Y_2$.

Thus, we have proved that, $D_{ev}(P_n^*, i) \subseteq Y_1 \bigcup Y_2$. Hence

$$D_{ev}(P_n^*, i) = \left\{ \begin{array}{l} \{X_1 \bigcup \{e_{2n-1}\} / X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\} \bigcup \\ \{X_2 \bigcup \{e_{2n-2}\} / X_2 \in D_{ev}(P_{n-1}^*, i-1)\} \end{array} \right\}$$

iv) Let $Y_1 = \{X_1 \bigcup \{e_{2n-1}\} / X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\}$, $Y_2 = \{X_2 \bigcup \{e_{2n-2}\} / X_2 \in D_{ev}(P_{n-1}^*, i-1)\}$ and $Y_3 = \{X_3 \bigcup \{e_{2n-2}\} / X_3 \in D_{ev}(P_{n-2}^*, i-1)\}$.

Obviously, $Y_1 \bigcup Y_2 \bigcup Y_3 \subseteq D_{ev}(P_n^*, i)$. Now, let $Y \in D_{ev}(P_n^*, i)$ then e_{2n-1} or $e_{2n-2} \in Y$. If $e_{2n-1} \in Y$, then we can write $Y = X_1 \bigcup \{e_{2n-1}\}$, for some $X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)$, that is $Y \in Y_1$. If $e_{2n-1} \notin Y$ and $e_{2n-2} \in Y$, then we can write $Y = X_2 \bigcup \{e_{2n-2}\}$ or $Y = X_3 \bigcup \{e_{2n-2}\}$, for some $X_2 \in D_{ev}(P_{n-1}^*, i-1)$ and $X_3 \in D_{ev}(P_{n-2}^*, i-1)$, that is $Y \in Y_2$ and Y_3. Thus, we have proved that, $D_{ev}(P_n^*, i) \subseteq Y_1 \bigcup Y_2 \bigcup Y_3$.

Hence

$$D_{ev}(P_n^*, i) = \left\{ \begin{array}{l} \{X_1 \bigcup \{e_{2n-1}\} / X_1 \in D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\} \bigcup \\ \{X_2 \bigcup \{e_{2n-2}\} / X_2 \in D_{ev}(P_{n-1}^*, i-1)\} \bigcup \\ \{X_3 \bigcup \{e_{2n-2}\} / X_3 \in D_{ev}(P_{n-2}^*, i-1)\} \end{array} \right\}$$

3. Edge-vertex domination polynomials of centipedes

Let $D_{ev}\left(P_n^*, x\right) = \sum_{i=\left\lceil \frac{n}{2} \right\rceil}^{n} d_{ev}\left(P_n^*, i\right) x^i$ be the edge-vertex domination polynomials of a centipede P_n^*. In this section, we derive the expression for $D_{ev}\left(P_n^*, x\right)$.

Theorem 3.1[8]

i) If $D_{ev}\left(P_n^*, i\right)$ is the family of edge-vertex dominating sets with cardinality i of P_n^*, then $\left|D_{ev}(P_n^*, i)\right| = \left|D_{ev}(P_n^* - \{e_{2n-1}\}, i-1)\right| + \left|D_{ev}(P_{n-1}^*, i-1)\right| + \left|D_{ev}(P_{n-2}^*, i-1)\right|$.

ii) For $n = 3$,

$$D_{ev}(P_n^*, x) = x\left[D_{ev}(P_n^* - \{e_{2n-1}\}, x) + D_{ev}(P_{n-1}^*, x) + D_{ev}(P_{n-2}^*, x)\right]$$

with initial values $D_{ev}(P_1^*, x) = x$,

$$D_{ev}(P_2^*, x) = x^3 + 3x^2 + x,$$

$$D_{ev}(P_3^* - \{e_5\}, x) = x^4 + 4x^3 + 5x^2 + x.$$

Proof:

i) Using (i), (ii), (iii) and (iv) of theorem 2.5, we prove (i) part. Suppose, $D_{ev}(P_n^*-\{e_{2n-1}\},i-1)\neq\Phi$, $D_{ev}(P_{n-1}^*,i-1)=\Phi$ and $D_{ev}(P_{n-2}^*,i-1)=\Phi$ then $D_{ev}(P_n^*,i)=\{[e_{2n-1}]\}$. Therefore, $|D_{ev}(P_n^*,i)|=|\{[e_{2n-1}]\}|=1$.

In this case $|D_{ev}(P_n^*-\{e_{2n-1}\},i-1)|=|\{[e_{2n-1}]\}|=1$ and $|D_{ev}(P_{n-1}^*,i-1)|=|D_{ev}(P_{n-2}^*,i-1)|=0$.

Therefore, in this case the theorem holds.

Suppose $D_{ev}(P_n^*-\{e_{2n-1}\},i-1)=D_{ev}(P_{n-1}^*,i-1)=\Phi$ and $D_{ev}(P_{n-2}^*,i-1)\neq\Phi$ then $D_{ev}(P_n^*,i)=\{X\cup\{e_{2n-1}\}\,/\,X\in D_{ev}(P_n^*-\{e_{2n-1}\},i-1)\}$.

In this case $|D_{ev}(P_n^*-\{e_{2n-1}\},i-1)|=|D_{ev}(P_{n-1}^*,i-1)|=0$ and $|D_{ev}(P_{n-2}^*,i-1)|=1$. Therefore, $|D_{ev}(P_n^*,i)|=1+|D_{ev}(P_{n-1}^*,i-1)|+|D_{ev}(P_{n-2}^*,i-1)|$.

Therefore, $|D_{ev}(P_n^*,i)|=1$. Therefore,

$$|D_{ev}(P_n^*,i)|=|D_{ev}(P_n^*-\{e_{2n-1}\},i-1)|+|D_{ev}(P_{n-1}^*,i-1)|+|D_{ev}(P_{n-2}^*,i-1)|.$$

Suppose $D_{ev}(P_n^*-\{e_{2n-1}\},i-1)=D_{ev}(P_{n-1}^*,i-1)\neq\Phi$ and $D_{ev}(P_{n-2}^*,i-1)=\Phi$. In this case we have,

$$D_{ev}(P_n^*,i)=\left\{\begin{array}{l}\{X_1\cup\{e_{2n-1}\}\,/\,X_1\in D_{ev}(P_n^*-\{e_{2n-1}\},i-1)\}\cup\\ \{X_2\cup\{e_{2n-2}\}\,/\,X_2\in D_{ev}(P_{n-1}^*,i-1)\}\end{array}\right\}$$

by theorem 2.5(iii). Therefore,

$$|D_{ev}(P_n^*,i)|=|D_{ev}(P_n^*-\{e_{2n-1}\},i-1)|+|D_{ev}(P_{n-1}^*,i-1)|+|D_{ev}(P_{n-2}^*,i-1)|$$

.Suppose $D_{ev}(P_n^*-\{e_{2n-1}\},i-1)=D_{ev}(P_{n-1}^*,i-1)\neq\Phi$ and $D_{ev}(P_{n-2}^*,i-1)\neq\Phi$.

In this case we have,

$$D_{ev}(P_n^*,i)=\left\{\begin{array}{l}\{X_1\cup\{e_{2n-1}\}\,/\,X_1\in D_{ev}(P_n^*-\{e_{2n-1}\},i-1)\}\cup\\ \{X_2\cup\{e_{2n-2}\}\,/\,X_2\in D_{ev}(P_{n-1}^*,i-1)\}\cup\\ \{X_3\cup\{e_{2n-2}\}\,/\,X_3\in D_{ev}(P_{n-2}^*,i-1)\}\end{array}\right\}$$

by theorem 2.5 (iv). Therefore,

$$|D_{ev}(P_n^*,i)|=|D_{ev}(P_n^*-\{e_{2n-1}\},i-1)|+|D_{ev}(P_{n-1}^*,i-1)|+|D_{ev}(P_{n-2}^*,i-1)|$$

.

Hence the theorem.

ii) $d_{ev}(P_n^*,i)=d_{ev}(P_n^*-\{e_{2n-1}\},i-1)+d_{ev}(P_{n-1}^*,i-1)+d_{ev}(P_{n-2}^*,i-1)$.

$$\Sigma d_{ev}(P_n^*,i)x^i=\Sigma d_{ev}(P_n^*-\{e_{2n-1}\},i-1)x^i+\Sigma d_{ev}(P_{n-1}^*,i-1)x^i+\Sigma d_{ev}(P_{n-2}^*,i-1)x^i$$

$$\Sigma d_{ev}(P_n^*,i)x^i=x\left[\Sigma d_{ev}(P_n^*-\{e_{2n-1}\},i-1)x^{i-1}+\Sigma d_{ev}(P_{n-1}^*,i-1)x^{i-1}+\Sigma d_{ev}(P_{n-2}^*,i-1)x^{i-1}\right]$$

$$D_{ev}(P_n^*,i)x^i=x\left[D_{ev}(P_n^*-\{e_{2n-1}\},x)+D_{ev}(P_{n-1}^*,x)+D_{ev}(P_{n-2}^*,x)\right]$$

With initial values $D_{ev}(P_1^*,x)=x$,

$D_{ev}(P_2^*,x)=x^3+3x^2+x$,

$D_{ev}(P_3^*-\{e_5\},x)=x^4+4x^3+5x^2+x$.

Using theorem 3.1, we obtain $d_{ev}(P_n^*,i)$ for $1\le n\le 6$ and $1\le i\le 11$ as shown in **Table 1**.

Table 1. $d_{ev}(P_n^*,i)$ the number of edge-vertex dominating set of P_n^* with cardinality *i*.

i / n	1	2	3	4	5	6	7	8	9	10	11
P_1^*	1										
$P_2^*-\{e_3\}$	2	1									
P_2^*	1	3	1								
$P_3^*-\{e_5\}$	1	5	4	1							
P_3^*	0	3	8	5	1						
$P_4^*-\{e_7\}$	0	4	14	14	6	1					
P_4^*	0	1	10	23	19	7	1				
$P_5^*-\{e_9\}$	0	1	14	41	47	27	8	1			
P_5^*	0	0	5	32	69	67	34	9	1		
$P_6^*-\{e_{11}\}$	0	0	6	47	124	155	108	44	10	1	
P_6^*	0	0	1	21	102	212	229	143	53	11	1

In the following theorem, we obtain some properties of $d_{ev}(P_n^*,i)$.

Theorem 3.2

The following properties hold for the coefficients of $D_{ev}(P_n^*,x)$;

1) $d_{ev}(P_n^*,2n)=0$, for every $n\in N$.
2) $d_{ev}(P_n^*-\{e_{2n-1}\},2n-2)=1$, for every $n\in N$.
3) $d_{ev}(P_n^*,2n-1)=1$, for every $n\in N$.
4) $d_{ev}(P_n^*-\{e_{2n-1}\},2n-3)=2n-2$, for every $n\ge 2\in N$.
5) $d_{ev}(P_n^*,2n-2)=2n-1$, for every $n\ge 2\in N$.
6) $d_{ev}(P_n^*-\{e_{2n-1}\},2n-4)=(2n-1)(n-2)$, for every $n\ge 3\in N$.
7) $d_{ev}(P_n^*,2n-3)=(n-1)(2n-1)-2$, for every $n\ge 3\in N$.

Proof:

1) Proof of (1) is obvious. That is $d_{ev}(P_n^*, 2n) = 0$, for every $n \in N$.

2) Since $d_{ev}(P_n^* - \{e_{2n-1}\}, 2n-2) = \{e_1, e_2, e_3, ..., e_{2n-2}\}$ we have, $d_{ev}(P_n^* - \{e_{2n-1}\}, 2n-2) = 1$, for every $n \in N$.

3) Since $d_{ev}(P_n^*, 2n-1) = \{e_1, e_2, e_3, ..., e_{2n-1}\}$ we have, $d_{ev}(P_n^*, 2n-1) = 1$.

4) Proof of (4) follows from the **Table 1**.

5) By induction on *n*, the result is true for *n*=2.

LHS = $d_{ev}(P_4^*, 6) = 7$ (From Table 1). RHS = (2 x 4 – 1) = 7. Therefore, the result is true for *n*=2. Now suppose that the result is true for all numbers less than *n* and we prove it for *n*. By theorem 3.1,

$$d_{ev}(P_n^*, 2n-2) = d_{ev}(P_n^* - \{e_{2n-1}\}, 2n-3) + d_{ev}(P_{n-1}^*, 2n-3) + d_{ev}(P_{n-2}^*, 2n-3)$$

$$d_{ev}(P_n^*, 2n-2) = (2n-2) + 1 + 0 = 2n-1$$

.6) Proof of (6) follows from the Table 1.

7) By induction on *n*, the result is true for *n*=3. LHS = $d_{ev}(P_3^*, 3) = 8$ (From Table 1). RHS = (3 – 1) (6 – 1) – 2 = 8. Therefore, the result is true for *n*=3. Now suppose that the result is true for all numbers less than *n* and we prove it for *n*. By theorem 3.1,

$$d_{ev}(P_n^*, 2n-3) = d_{ev}(P_n^* - \{e_{2n-1}\}, 2n-4) + d_{ev}(P_{n-1}^*, 2n-4) + d_{ev}(P_{n-2}^*, 2n-4)$$

$$= (2n-1)(n-2) + 2(n-1) - 1 + 0$$

$$= (2n-1)(n-2) + (2n-1) - 2$$

$$= (2n-1)(n-2+1) - 2$$

$$= (2n-1)(n-1) - 2$$

4. Conclusion

In [9], the domination polynomial of path was studied and obtained the very important property, $d(P_n, i) = d(P_{n-1}, i-1) + d(P_{n-2}, i-1) + d(P_{n-3}, i-1)$

It is interesting that we have derived an analogues relation for the Edge-Vertex domination of centipede of the form,

$$d_{ev}(P_n^*, i) = d_{ev}(P_n^* - \{e_{2n-1}\}, i-1) + d_{ev}(P_{n-1}^*, i-1) + d_{ev}(P_{n-2}^*, i-1)$$

One can characterise the roots of the polynomial $D_{ev}(P_n^*, x)$ and identify whether they are real or complex. Another interesting character to be investigated is whether $D_{ev}(P_n^*, x)$ is log concave or not.

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