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RESEARCH ARTICLE

# Geometric decomposition of complete tripartite graphs

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## Abstract

Let $G = (V, E)$ be a simple connected graph with $p$ vertices and $q$ edges. If $G_1, G_2, G_3, \ldots, G_n$ are connected edge disjoint subgraphs of $G$ with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, G_3, \ldots, G_n)$ is said to be a decomposition of $G$. A decomposition $(G_1, G_2, G_3, \ldots, G_n)$ of $G$ is said to be an Arithmetic Decomposition if each $G_i$ is connected and $|E(G_i)| = a + (i-1)d$, for every $i = 1, 2, 3,\ldots,n$ and $a,d \in$ N. In this paper, we introduced a new concept Geometric Decomposition. A decomposition $(G_a, G_{ar}, G_{ar^2}, G_{ar^3}, \ldots, G_{ar^{n-1}})$ of $G$ is said to be a Geometric Decomposition (GD) if each $G_{ar^{i-1}}$ is connected and $|E(G_{ar^{i-1}})| = ar^{i-1}$, for every $i = 1, 2, 3,\ldots,n$ and $a, r \in$ N. Clearly $q = \frac{a(r^n-1)}{r-1}$. If $a = 1$ and $r = 2$, then $q = 2^n - 1$. Also we obtained necessary and sufficient conditions of complete tripartite graphs which are admitting a Geometric Decomposition.

**Keywords**

Decomposition of graph
Complete tripartite graph
Arithmetic decomposition
Geometric decomposition

## Introduction

In this paper, we consider simple undirected graph without loops or multiple edges. For all other standard terminology and notations we follow Harary [1]. Gnanadhas and Paulraj Joseph introduced the concept of Continuous Monotonic Decomposition (CMD) of graphs[2]. Ebin Raja Merly and Gnanadhas introduced the concept of Arithmetic Odd Decomposition (AOD) of spider tree [3].

***Definition:*** Let $G = (V, E)$ be a simple connected graph with $p$ vertices and $q$ edges. If $G_1, G_2, G_3,\ldots, G_n$ are connected edge disjoint subgraphs of $G$ with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, G_3, \ldots, G_n)$ is said to be a decomposition of $G$.

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***Definition*:** A decomposition $(G_1, G_2, G_3, \ldots, G_n)$ of $G$ is said to be an Arithmetic Decomposition (AD) if each $G_i$ is connected and $|E(G_i)| = a+(i-1)d$, for every $i = 1, 2, 3, \ldots, n$ and $a, d \in$ N.

***Definition*:** A graph $G = (V, E)$ is $n$-partite, $n > 1$, if it is possible to partition $V$ into $n$ subsets: $V_1, V_2, V_3, \ldots, V_n$ such that every edge of $E$ joins a vertex of $V_i$ to a vertex of $V_j$, $i \neq j$. When $n = 3$, $n$-partite graphs are called complete tripartite graph.

### Geometric decomposition of graphs

***Definition*:** A decomposition $(G_a, G_{ar}, G_{ar^2}, G_{ar^3},\ldots,G_{ar^{n-1}})$ of $G$ is said to be a GD if each $G_{ar^{i-}}$ is connected and $|E(G_{ar^{i-1}})| = ar^{i-1}$, for every $i = 1, 2, 3, \ldots, n$ and $a, r \in \mathbb{N}$. Clearly $q = \frac{a(r^n - 1)}{r-1}$. If $a = 1$ and $r = 2$, then $q = 2^n - 1$. We know that $2^n - 1$ is the sum of $2^0, 2^1, 2^2, 2^3, \ldots, 2^{n-1}$. That is, $2^n - 1$ is the sum of $1, 2, 4, 8, \ldots, 2^{n-1}$. Thus we denote the GD as $(G_1, G_2, G_4, \ldots, G_{2^{n-1}})$. A Petersen graph admits GD $(G_1, G_2, G_4, G_8)$ of $G$.

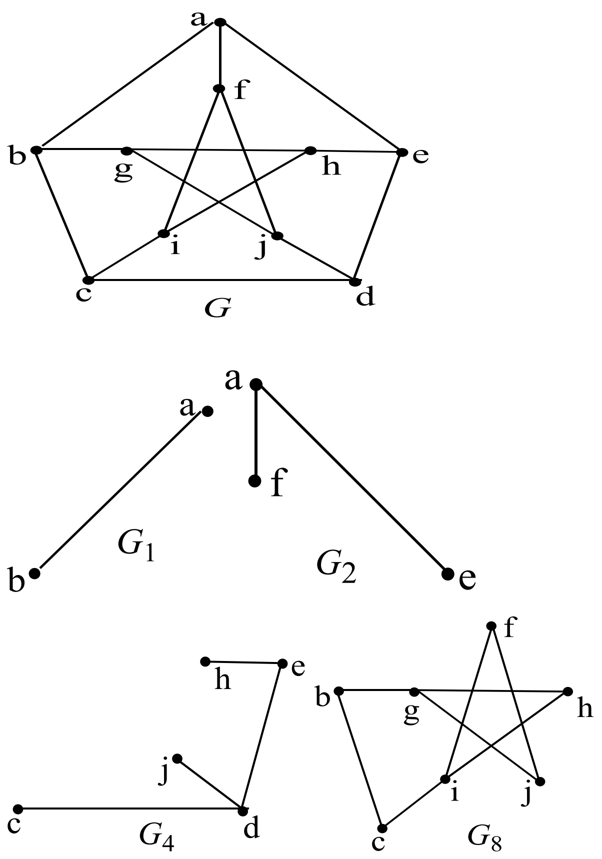



**Fig 1**

***Theorem*****:** A graph $G$ admits GD $(G_1, G_2, G_4, \ldots, G_{2^{n-1}})$ if and only if $q = 2^n - 1$ for each $n \in \mathbb{N}$.

***Proof*****:** Let $G$ be a connected graph with $q = 2^n - 1$. Let $u$, $v$ be two vertices of $G$ such that $d(u,v)$ is maximum.

Let $N_r(u) = \{v \in V / d(u,v) = r\}$. If $d(u) = 2^{n-1}$, choose $2^{n-1}$ edges incident with $u$. Let $G_{2^{n-1}}$ be a subgraph induced by these $2^{n-1}$ edges. If $d(u) < 2^{n-1}$, then choose $2^{n-1}$ edges incident with $u$ vertices of $N_1(u), N_2(u), \cdots$ successively such that the subgraph $G_{2^{n-1}}$ induced by these edges is connected. In both cases $G - G_{2^{n-1}}$ has a connected component $H_1$ with $2^n - 2^{n-1} - 1$ edges.

Now, consider $H_1$ and proceed as above to get $G_{2^{n-2}}$ such that $H_1 - G_{2^{n-2}}$ has a connected component $H_2$ of size $2^n - 2^{n-1} - 2^{n-2} - 1$ edges. Proceeding like this we get a connected subgraph $G_2$ such that $H_{2^{n-2}}$ is a graph with one edge taken as $G_1$. Thus $(G_1, G_2, G_4, \ldots, G_{2^{n-1}})$ is a GD of $G$. Conversely, Suppose $G$ admits GD $(G_1, G_2, G_4, \ldots, G_{2^{n-1}})$. Then obviously, $q(G) = 1+2+4+\ldots+2^{n-1} = 2^n - 1$ for each $n \in \mathbb{N}$.

**Geometric decomposition of complete tripartite graphs**

***Theorem*****:** A complete tripartite graph $K_{1,\, 2^n-1,\, 2^n-1}$ admits GD $(G_1, G_2, G_4, \ldots, G_{2^{2n-1}})$ if and only if $q = 2^{2n} - 1$.

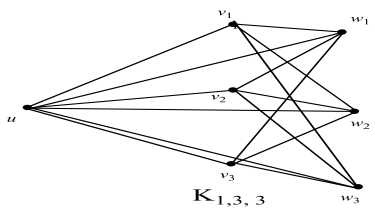
***Proof:*** Suppose $q(K_{1,\, 2^n-1,\, 2^n-1}) = (2^n-1)+(2^n-1)^2+(2^n-1) = 2^n-1+2^{2n}+1-2.2^n+2^n-1 = 2^{2n}-1$ for each $n$.

Applying induction on $n$. When $n = 1$, $q = 3$. Since $G$ is connected, $G$ can be decomposed into $G_1$ and $G_2$. When $n = 2$, $q = 15$, $G$ can be decomposed into $G_1$, $G_2$, $G_4$, and $G_8$.

Suppose the result is true for $n = k$. Let $G$ be any connected graph with $q = 2^{2k}-1$. Then $G$ can be decomposed into $G_1, G_2, G_4, \ldots, G_{2^{2k-1}}$. Let $n = k + 1$. Let $G'$ be a connected graph with $2^{2(k+1)} - 1$ edges. Now $2^{2(k+1)} - 1 = 2^{2k+2} - 1 = 4.2^{2k} - 1 = (2^{2k} - 1) + 3.\, 2^{2k}$.

Therefore $G'$ contains $(2^{2k} - 1) + 3.2^{2k}$ edges. Let $G''$ be a connected subgraph of $G'$ with $3.2^{2k}$ edges. Now $3.2^{2k} = 2^{2k} + 2^{2k+1}$. Therefore $G''$ can be decomposed in to $G_{2^{2k}}$ and $G_{2^{2k+1}}$. Let $G^*$ be another connected subgraph of $G'$ excluding the edges of $G_{2^{2k}}$ and $G_{2^{2k+1}}$. Clearly $G^*$ contains $2^{2k} - 1$ edges. By induction hypothesis $G^*$ can be decomposed into $k$-subgraphs $G_1, G_2, G_4, \ldots, G_{2^{2k-1}}$. Thus $G'$ can be decomposed into $G_1, G_2, G_4, \ldots, G_{2^{2k-1}}, G_{2^{2k}}$ and $G_{2^{2k+1}}$. That is, $G'$ admits GD having $(k+1)$ subgraphs. Hence by induction if $q = 2^{2n} - 1$ for each $n \in \mathbb{N}$, $G$ admits GD $(G_1, G_2, G_4, \ldots, G_{2^{2n-1}})$. Conversely, Suppose $G$ admits GD $(G_1, G_2, G_4, \ldots, G_{2^{2n-1}})$. Then obviously, $q(G) = 1+2+4+\ldots+2^{2n-1} = 2^{2n}-1$ for each $n \in \mathbb{N}$.

**Example:** The GD $(G_1, G_2, G_4, G_8)$ of $K_{1,3,3}$.



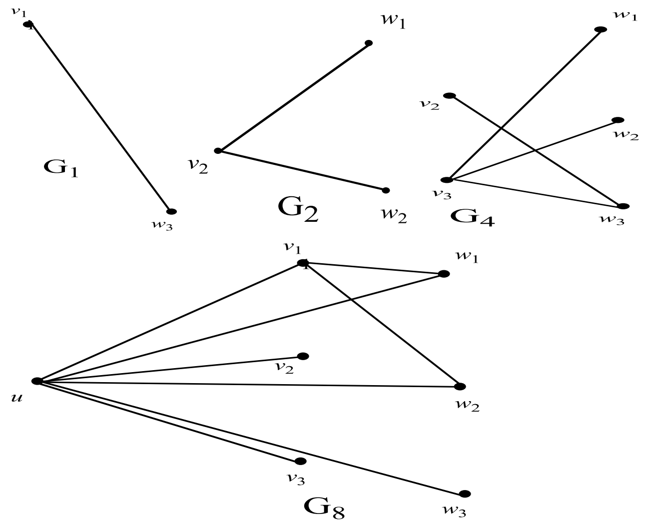



**Fig 2**

***Theorem*:** A complete tripartite graph $K_{1,3,m}$ accepts GD ($G_1, G_2, G_4, \ldots, G_{2^{n+1}}$) if and only if $m = 2^n - 1$.

***Proof*:** Assume that a complete tripartite graph $K_{1,3,m}$ accepts GD ($G_1, G_2, G_4, \ldots, G_{2^{n+1}}$). We have, $q(K_{1,3,m}) = 4m+3$. That is, the graph $K_{1,3,m}$ accepts GD ($G_1, G_2, G_4, \ldots, G_{2^{n+1}}$) if and only if $q(K_{1,3,m}) = 2^{n+2}-1$. That is, $4m+3=2^{n+2}-1$. That is, $m = 2^n - 1$. Conversely, Consider $K_{1,3,m}$ with $m = 2^n - 1$. Then $q(K_{1,3,m}) = 4(2^n - 1)+3 = 2^{n+2}-1$. This implies that, $K_{1,3,m}$ can be decomposed into $G_1, G_2, G_4, \ldots, G_{2^{n+1}}$.

**Example:** The GD ($G_1, G_2, G_4, G_8, G_{16}$) of $K_{1,3,7}$.

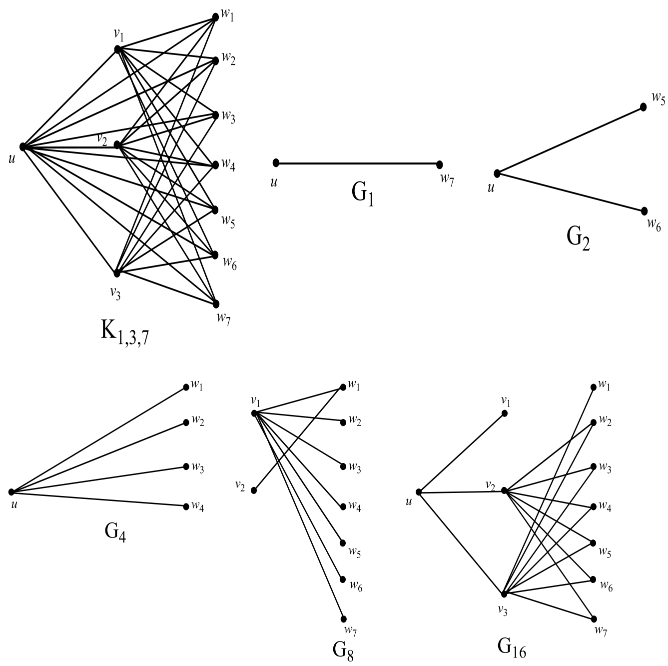



**Fig 3**

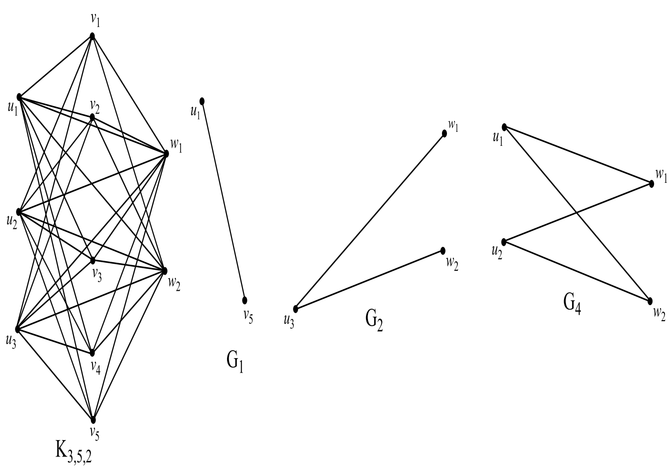
The following table shows that GD of $K_{1,3,m}$.

| $m$ | $q(K_{1,3,m})$ | GD |
|---|---|---|
| 1 | 7 | $G_1, G_2, G_4$. |
| 3 | 15 | $G_1, G_2, G_4, G_8$. |
| 7 | 31 | $G_1, G_2, G_4, \ldots, G_{16}$. |
| 15 | 63 | $G_1, G_2, G_4, \ldots, G_{32}$. |
| 31 | 127 | $G_1, G_2, G_4, \ldots, G_{64}$. |
| 63 | 255 | $G_1, G_2, G_4, \ldots, G_{128}$. |
| 127 | 511 | $G_1, G_2, G_4, \ldots, G_{256}$. |
| 255 | 1023 | $G_1, G_2, G_4, \ldots, G_{512}$. |
| 511 | 2047 | $G_1, G_2, G_4, \ldots, G_{1024}$. |
| 1023 | 4095 | $G_1, G_2, G_4, \ldots, G_{2048}$. |

***Theorem:*** A Complete Tripartite graph $K_{3,5,m}$ accepts GD ($G_1, G_2, G_4, \ldots, G_{2^{n+3}}$) if and only if $m = 2(2^n-1)$.

***Proof:*** Assume that a complete tripartite graph $K_{3,5,m}$ accepts GD ($G_1, G_2, G_4, \ldots, G_{2^{n+3}}$). We have, $q(K_{3,5,m}) = 8m+15$. That is, the graph $K_{3,5,m}$ accepts GD ($G_1, G_2, G_4, \ldots, G_{2^{n+3}}$) if and only if $q(K_{3,5,m}) = 2^{n+4}-1$.
That is, $8m+15 = 2^{n+4}-1$.
That is, $m = 2(2^n -1)$.
Conversely, Consider $K_{3,5,m}$ with $m = 2(2^n -1)$. Then $q(K_{3,5,m}) = 8(2(2^n -1))+15 = 2^{n+4} -1$. This implies that, $K_{3,5,m}$ can be decomposed into $G_1, G_2, G_4, \ldots, G_{2^{n+3}}$.

***Example:*** The GD($G_1, G_2, G_4, G_8, G_{16}$) of $K_{3,5,2}$.



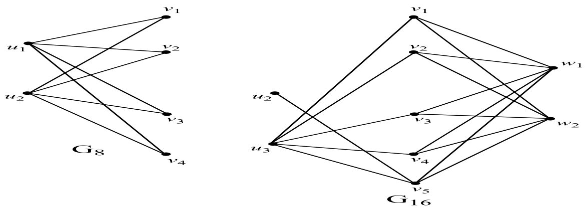



**Fig 4**

The following table gives the GD of $K_{3,5,m}$ for different values of *m*.

| $m$ | $q(K_{3,5,m})$ | GD |
|---|---|---|
| 2 | 31 | $G_1, G_2, G_4, \ldots, G_{16}$. |
| 6 | 63 | $G_1, G_2, G_4, \ldots, G_{32}$. |
| 14 | 127 | $G_1, G_2, G_4, \ldots, G_{64}$. |
| 30 | 255 | $G_1, G_2, G_4, \ldots, G_{128}$. |
| 62 | 511 | $G_1, G_2, G_4, \ldots, G_{256}$. |
| 126 | 1023 | $G_1, G_2, G_4, \ldots, G_{512}$. |
| 254 | 2047 | $G_1, G_2, G_4, \ldots, G_{1024}$. |
| 510 | 4095 | $G_1, G_2, G_4, \ldots, G_{2048}$. |
| 1022 | 8191 | $G_1, G_2, G_4, \ldots, G_{4096}$. |
| 2046 | 16383 | $G_1, G_2, G_4, \ldots, G_{8192}$. |

In the same manner complete tripartite graph $K_{2,5,m}$ accepts GD $(G_1, G_2, G_4, \ldots, G_{2^{3n+1}})$ if and only if $m = \frac{2^{3n+2}-11}{7}$.

## Conclusions

The GD of complete tripartite has been studied. The study can be extended for m-paratite graphs.

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